Thermal Imaging of Nanostructures by Quantitative Optical Phase Analysis

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P robing temperature at the nanoscale is fundamentally a complicated task mainly because heat has a nonpropagative nature, unlike light. Yet, the ability to investigate thermal-induced phenomena on the nanoscale would help in the development of unprecedented applications in many active current areas of research,† namely, photothermal cancer therapy,2,3 drug release,4 nanosurgery,5 opto-acoustic imaging,6,7 nanochemistry,8,9 microfluidics,10 magnetic recording11 and thermonics.12 Twenty years ago, as a temperature microscopy technique, the use of a sharp hybrid tip as a nanothermocouple was proposed.13,14 Even though such an approach achieved a spatial resolution of 50 nm, it appeared to be very invasive due to the proximity of the tip to the sample. More recently, several thermal microscopy techniques have been developed that are much less invasive since they are based on far-field optical measurements. These optical techniques rely either on fluorescence intensity,15,16 fluorescence polarization anisotropy,17,18 fluorescence spectroscopy,19–21 Raman spectroscopy,22 infrared spectroscopy,23 or X-ray spectroscopy24 measurements. However, all these techniques suffer from drawbacks that make them suited only for particular cases. Namely, they suffer from a slow read-out rate,19–22 a low resolution,23 a lack of reliability,15,16 a poor temperature sensitivity,22 or the necessity to modify (tag) the medium with fluorescent molecules or nanocrystals.15–21

In this paper, we propose a thermal microscopy technique sensitive to the thermal-induced variation of the refractive index of the medium of interest. This optical technique circumvents the limitations of all the previous techniques mentioned above and is straightforward to implement on any conventional optical microscope. We explain how it leads to a quantitative real-time retrieval of the distributions of both the temperature and the source of heat with a submicrometric resolution. The theory and the retrieval algorithms are detailed. We chose to illustrate the capabilities of this technique on gold nanoparticles (NPs), which act as ideal nanosources of heat when illuminated at their plasmonic resonance frequency. Moreover, we show how a quantitative measurement of the actual absorption cross section can be achieved, whatever the nature of the absorbing structure. We chose to name this technique TIQSI for thermal imaging using quadrature shear interferometry.

ABSTRACT

We introduce an optical microscopy technique aimed at characterizing the heat generation arising from nanostructures, in a comprehensive and quantitative manner. Namely, the technique permits (i) mapping the temperature distribution around the source of heat, (ii) mapping the heat power density delivered by the source, and (iii) retrieving the absolute absorption cross section of light-absorbing structures. The technique is based on the measurement of the thermal-induced refractive index variation of the medium surrounding the source of heat. The measurement is achieved using an association of a regular CCD camera along with a modified Hartmann diffraction grating. Such a simple association makes this technique straightforward to implement on any conventional microscope with its native broadband illumination conditions. We illustrate this technique on gold nanoparticles illuminated at their plasmonic resonance. The spatial resolution of this technique is diffraction limited, and temperature variations weaker than 1 K can be detected.

KEYWORDS: microscopy · thermodynamics · phase imaging · plasmonics · gold nanoparticles
RESULTS AND DISCUSSION

We consider in this work a two-dimensional distribution of absorbers (gold nanoparticles) distributed at the interface between a solid (glass substrate) and a liquid environment (like water). When the absorbers are illuminated, heat is generated and a temperature steady state profile appears over a time scale that depends on the spatial extension of the source of heat (typically below 10 μs for structures smaller than 1 μm, in water). This temperature increase is responsible for a variation of the refractive index of the surrounding liquid over a typical length much larger than the size of the particle itself. This effect has been recently exploited independently by the groups of Lounis and Orrit as a localization technique of NPs or single molecules. NPs and molecules of interest in these works were nonluminescent and far too small to scatter any incident light, making them invisible using any conventional optical technique. To circumvent this limitation, the idea developed in these groups was to exploit the light absorption capabilities of the nano-objects to thermally induce an extended variation of the refractive index of the surroundings, much easier to detect than the nano-objects themselves. However, this localization technique is based on the modulation of the heating laser to achieve an heterodyne detection. For this reason, it cannot be used to measure any steady-state temperature distribution. Moreover, this technique remains slow and requires a complex experimental configuration comprising an acousto-optical modulator and a heterodyne detector.

Recently, our group has developed an optical phase microscopy technique capable of mapping quantitatively the optical path difference (OPD) through a refractive object in real time with a sensitivity of 1 nm. This technique was illustrated by imaging microscopic dielectric beads and living organisms. The experimental approach is based on the use of a quadriwave lateral shearing interferometer as a wavefront analyzer (WFA) (see Supporting Information for more details). Basically, it consists of a modified Hartmann grating (MHG) associated with a regular CCD camera. The device, named Sid4Bio, was purchased from Phasics SA. The interferogram produced by the MHG and recorded by the CCD camera can be processed in real time to retrieve the OPD. Interestingly, this interferometric technique is achromatic and can be mounted on any conventional white-light transmission microscope. In the present work, we demonstrate how this experimental approach can be the basis of an efficient and easy-to-implement thermal imaging technique. The experimental setup is detailed in Figure 1. All the samples we used consist of a sandwich structure where a water layer of thickness \( h \) is limited by two glass coverslips in the \( z \) direction. The gold nanoparticles acting as nanosources of heat are lying on the bottom coverslip.

When experiencing a temperature variation \( \delta T(r) = T(r) - T_{\infty} \), a given medium undergoes a variation \( \delta n(r) \) of its refractive index. This variation is usually described by a Taylor development of the \( N \)th order:

\[
\delta n = \sum_{j=0}^{N} \beta_j \delta T^j
\]

where \( \beta_j \) are empirical parameters (see Supporting Information or ref 32 for values). For the sake of simplicity, we shall consider in this theoretical part only the first (linear) term of this Taylor series: \( \delta n = \beta_1 \delta T \). In water, this assumption is valid if the temperature increase over the ambient temperature does not exceed \( \sim 20 \degree \text{C} \). It makes the equations linear and simplifies the algorithm used to retrieve the temperature profile. When higher temperature increases are considered, a nonlinear iterative numerical procedure is required. The approach is more complex but still simple to implement as detailed in Supporting Information.

To begin with, let us consider a point-like source of heat located at the solid—liquid planar interface at \( (x,y,z) = (0,0,0) \). In the surrounding liquid, the temperature
distribution is governed by the Poisson equation:

\[ \kappa \nabla^2 T(r) = P_0 \delta(t) \]  \hspace{1cm} (2)

where \( \kappa \) is the thermal conductivity of the surrounding liquid (0.6 W/m/K for water), \( T(r) \) the temperature and \( P_0 \) the power absorbed (i.e., delivered) by the point-like source of heat. In spherical coordinates, the solution reads

\[ T(r) = T_0 + \frac{P_0}{4\pi \kappa r} = T_0 + \frac{P_0}{r} \]  \hspace{1cm} (3)

\[ T(r) = T_0 + \tilde{P}_0 G_T(r) \]  \hspace{1cm} (4)

where \( T_0 \) is the ambient temperature, \( P_0 = P_0/(4\pi \kappa) \) is the normalized power, and \( G_T(r) = 1/r \) the standard Green’s function associated to the Poisson equation.

Experimentally, we use the WFA to measure two phase images of a given area: One reference image acquired (once far all) without heating and one image under heating. The subtraction of these two images provides a measure of the spatial distribution of the optical phase difference \( \delta/(x, y) \) through the liquid layer specifically due to thermal-induced variation of the refractive index distribution \( \delta n(x, y, z) \). It can be written as

\[ \delta/(x, y) = \int_0^h \delta n(t) \, dz = \int_0^h \beta_i \delta T(r) \, dz \]  \hspace{1cm} (5)

According to eq 3, the profile of the temperature variation \( \delta T \) reads in radial coordinates \((r, z)\):

\[ \delta T(r, z) = \frac{P_0}{\sqrt{r^2 + z^2}} \]  \hspace{1cm} (6)

Using eqs 5 and 6 and after integration, the OPD reads

\[ \delta/(\rho) = P_0 \beta_i \ln(h/\rho + \sqrt{1 + (h/\rho)^2}) \]  \hspace{1cm} (7)

\[ \delta/(\rho) = P_0 \beta_i \sinh^{-1}(h/\rho) = P_0 G_r(\rho) \]  \hspace{1cm} (8)

where \( G_r(\rho) = \beta_i \sinh^{-1}(h/\rho) \) stands for the Green’s function for the phase distribution, since the source of heat is a Dirac distribution. By eliminating the radial coordinate \( \rho \) in eqs 6 and 7, one ends up with a bijective relation between the optical phase difference and the temperature increase at \( z = 0 \):

\[ \delta T(\rho, 0) = \frac{P_0}{\tilde{R}} \sinh \left( \frac{\delta/(\rho)}{P_0 \beta_i} \right) \]  \hspace{1cm} (9)

Note that this formula is only valid for a point-like source of heat, that is, a gold disk, 300 nm in diameter and 40 nm thick illuminated by a laser beam at \( \lambda_0 = 808 \text{ nm} \) (Figure 2a). Figure 2b displays the raw image of the OPD measured by the WFA. From this image, we have applied the Tikhonov algorithm to retrieve the actual HPD and temperature distributions, as displayed in Figure 2c.d. Interestingly, the temperature extension (Figure 2d) is much narrower than the extension of the OPD distribution. This observation is consistent with the \(-\ln(\rho)\) decrease of the OPD Green’s function (eq 7), compared to the much steeper \(1/\rho\) decrease of the temperature Green’s function (eq 4). In this measurement, 30 frames...
of 100 ms were averaged. The standard deviation of the noise in the raw OPD image (Figure 2b) is 0.15 nm and in the temperature image (Figure 2d), 1.1 K.

In experimental optics, determining the actual absorption cross section (ACS) of a unique absorber is not straightforward mainly because the absorption process is precisely not associated with any light reemission that could contain information and be detected in the far-field (unlike extinction or scattering cross sections). Optical techniques to measure ACS have been proposed recently. Nevertheless, all of them suffer from at least one of these drawbacks: (i) They consist in fact in measuring extinction instead of absorption. This assimilation is valid only for small particles, for instance, when scattering is negligible (extinction = absorption + scattering). (ii) They do perform absorption spectroscopy, but they do not yield a quantitative measurement of the cross section. In this second case, a quantitative ACS can be eventually estimated indirectly by comparing the photothermal signal with the one obtained on a reference absorber with a supposedly known ACS. Our approach circumvents these two limitations at the same time: It allows the acquisition of a direct quantitative measurement of the actual absorption cross section \( \sigma_{\text{abs}} \) without any assumption and independently of the nature of the absorber. One just needs to estimate the absorbed power \( P_0 \) and then calculate \( \sigma_{\text{abs}} = P_0/l \) where \( l \) is the irradiance of the laser beam (power per unit area). There are two possibilities to estimate \( P_0 \). The first one consists in fitting the radial profile of the OPD image (as shown in Figure 2e) using equation 7 with \( P_0 \) (and eventually \( h \)) as the unknown parameter. This first approach gives in the case of Figure 2f: \( \sigma_{\text{abs}} = 14930 \pm 530 \, \text{nm}^2 \). The second approach consists in estimating \( P_0 \) by a spatial integration of the HPD image (Figure 2c). It naturally yields a very similar result: \( \sigma_{\text{abs}} = 15300 \pm 600 \, \text{nm}^2 \). Calculations using the Green’s dyadic tensor method on a gold disk, 288 nm in diameter, 39 nm thick, discretized using 2066 cells and illuminated at \( \lambda = 808 \, \text{nm} \), led to \( \sigma_{\text{abs}} = 17200 \, \text{nm}^2 \), which is the good order of magnitude. The difference of 14% with the experimental measurement can be mainly explained by the fact that the real structure is not a perfect disk. The accuracy of the measurement can be optimized by acquiring a set of images at different irradiances, as illustrated in Figure 2f. The first approach is handy since it does not require the computation of the HPD distribution, while the second approach is more general and applies even for extended (not point-like) sources of heat. Note that one does not need to optically resolve the NP to measure its ACS. Indeed, the ACS retrieval procedure is based on the fit of the logarithmic OPD profile around the particle that extends to infinity, even for nanometric/point-like particles (see Figure 2a–c).

We shall now present experimental results that address the situation of an extended distribution of heat sources. In Figure 3, experiments have been carried out on an array of quasi-hexagonally organized NPs. The sample has been done by block-copolymer lithography. It consists of spherical gold NPs, \( d = 30 \pm 3 \, \text{nm} \) in diameter and separated by \( \rho = 100 \pm 10 \, \text{nm} \). The NP interdistance is small enough compared to the resolution of the microscope to consider the HPD as a continuous physical quantity. The heat is generated by a laser beam at \( \lambda_0 = 532 \, \text{nm} \), close to the resonance wavelength of the gold particles. In the first case, the NP array covers the whole sample (Figure 3a) and the laser illuminates uniformly a circular area of diameter 8.0 \( \mu \text{m} \). In the second case, NP distributions form cross-like structures (Figure 3f). One of them is uniformly illuminated by the heating laser. From the optical path difference measured by the WFA (Figure 3b–g) the distribution of the HPD is computed using the Tikhonov inversion algorithm. As expected, the heat source distributions (Figure 3c–h) are well delimited while the temperature distributions tend to spread out (Figure 3d–i). To verify the consistency of the temperature distribution, we performed numerical simulation of the temperature profile in Figure 3d using the Green’s function formalism (Figure 3e). The input

Figure 2. (a) Scanning electronic microscope (SEM) image of the gold structure under study. This structure was illuminated at a wavelength \( \lambda_0 = 808 \, \text{nm} \). (b) Raw image of \( P_0(x,y) \) \( T = 12.8 \, \text{mW}/\text{cm}^2 \). (c) Heat power density \( P_0(x,y) \) processed from image b using the NL algorithm. (d) Temperature distribution \( \delta T(x,y) \) processed from image c using equation 9 (dashed line). (f) Delivered power \( P_0 \) as function of the laser irradiance, \( \rho(x,y) \), fitted using equation 9 with \( P_0 \) and \( h \) as the unknown parameters.
parameters regarding the geometry and the illumination conditions were $I = 10^5 \mu W/\mu m^2$, $D = 8.0 \mu m$, $p = 105 \text{ nm}$, $d = 28 \text{ nm}$. The only variable parameters were $d$ and $p$ which remain very close to the measured values on the SEM image. The matching is very good as observed in Figure 3e, which reinforces the validity of our postprocessing numerical model. A last result that we wish to share is presented in Figure 3j. It shows that, as expected, the temperature increase is proportional to the laser irradiance, even over a wide range of temperature. The nonlinearities of the problem are thus well taken into account in our numerical procedure (see Supporting Information).

In this last paragraph, we wish to discuss the temperature sensitivity that the TIQSI method can achieve. Figure 4 presents a measurement performed on an extended quasi-hexagonal array of nanodots. The structure is illuminated by a uniform circular beam of diameter $D = 8.0 \mu m$ and uniform irradiance $10^5 \mu W/\mu m^2$ at $\lambda = 532 \text{ nm}$; such a low irradiance ($I = 3.7 \mu W/\mu m^2$) led to a fairly noisy OPD image (Figure 4a) due to a weak temperature increase. Processing of the OPD image led to a temperature image presented in Figure 4b and revealed that the temperature increase was about $1 \text{ K}$. One can see that such a temperature increase is perfectly visible on the OPD image, even though it leads to a very noisy temperature image. The standard deviation of the noise in the OPD image (Figure 4a) is $0.044 \text{ nm}$ and in the temperature image (Figure 4b), $0.36 \text{ K}$. The higher noise level in the temperature image compared with the OPD image is inherent to the deconvolution procedure, and mostly features low frequencies. Below a temperature increase of $1 \text{ K}$, it is hard to obtain a well-defined temperature map. However, since most of the noise observed on the interferogram recorded by the CCD image is due to dark current, it could be possible to increase the signal-to-noise ratio and reach even better temperature sensitivity by increasing the number of accumulated frames, or by using a better cooled CCD camera. In this work, we chose a water environment in order to match the usual conditions of the potential applications, but another way to improve the temperature sensitivity could be to use a liquid medium featuring a higher $\beta_1$ value like alcohols or alkanes. Note that no physical limitation exists on the temperature accuracy, unlike the spatial resolution that is bound to remain diffraction-limited.

CONCLUSION

We introduced a thermal microscopy technique sensitive to thermal-induced refractive index variation.
The measurement is achieved using a simple experimental configuration consisting of a Hartmann diffraction grating attached to a CCD camera. This technique, which we named TIOSI, cumulates many valuable advantages compared with previous thermal imaging techniques: (i) It is fast. According to the CCD frame rate, it takes around one second to acquire an acceptable image in most cases. (ii) No scanning is required. It is a one-shot measurement of a 2D image. (iii) It has a diffraction limited resolution (less than 500 nm). (iv) No modification of the sample is required, like using fluorescent markers. It works whatever the sample as far as there is a liquid environment. (v) No need to use sophisticated devices such as heterodyne detection, acousto-optic modulator, and spectrometer, etc., like previous thermal imaging techniques. All the thermal measurements can be performed by plugging in the CCD/grating device on a conventional optical microscope, just like a regular CCD camera. (vi) The TIOSI technique features an unprecedented versatility since it can quantitatively measure temperature, heat source density, and absorption cross sections of nanoparticles and nanostructures. We chose to illustrate the abilities of the technique on gold NPs, which stand for ideal nanosources of heat. However, we wish to emphasize that the method is not restricted to metal NPs and can be extended to any 2D-distribution of heat sources, independent of their nature. Moreover, this technique is straightforward to implement on any conventional microscope. For these reasons, pending important applications are envisioned not only in plasmonics but also in areas such as nanoelectronics, microfluidics, or cell biology.

METHODS

Fabrication of the Quasi-hexagonal Arrays of Gold Nanodots. In a typical synthesis, polystyrene(1056)-block-poly(2-vinylpyridine)-(495) (PS1056-b-P2VP495) from Polymer Source Inc. was dissolved at room temperature in anhydrous toluene (Sigma-Aldrich) with a concentration of 5 mg/mL and stirred for 2 days. The quantity of gold precursor was calculated relative to the number of P2VP units with a loading parameter equal to 0.5, that is, 1 molecule of HAuCl₄ for 2 P2VP monomers. Hydrogen tetrachloroaurate (III) trihydrate (HAuCl₄·3H₂O, Sigma-Aldrich) was added to the dBCP solution and stirred for 2 days in a sealed glass vessel. Glass coverslips (Carl Roth) were cleaned in a piranha solution for at least 5 h and were extensively rinsed with Milli-Q water and dried under a stream of nitrogen. Micellar monolayers were prepared by dip-coating a glass coverslip into the previously prepared solutions with a concentration of 5 mg/mL and stirred for 2 days. Fourier transform infrared (FTIR) spectra and thermogravimetric analysis (TGA) of the micropatterned arrays of gold nanodots, a 15 nm thick layer of AR-PS350 photore sist (Alfresist GmbH) was spin-coated onto the arrays. Illumination was performed under a maskaligner (SUSS Microtec GmbH) with a dose of 175 mJ/cm² from a HBR 350 mercury lamp, and the microstructures were developed according to the manufacturer’s instructions. The enlargement of gold nanodots was performed in an aqueous solution containing hydroxylamine (0.02 mM) and HAuCl₄·0.1 wt % for 10 min. Scanning electron measurements were performed with a Dual BeamTM (FIB/SEM) instrument (Quanta 3D FEG, FEI, Hillsboro).

Conflict of Interest: The authors declare the following competing financial interest: PB was a PhD student from September 2010 to December 2011 with a financial support that came partly from the Phasics company.

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Supporting Information Available: Additional figures and calculations as described in the text. This material is available free of charge via the Internet at http://pubs.acs.org.

REFERENCES AND NOTES

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Supplementary information

A. Linear deconvolution numerical method

The optical index $n$ of a material depends on the temperature $T$. This dependency can be approximated by a Taylor development such as:

$$n(T) = \sum_{j=0}^{M} b_j T^j$$

with $T$ in degree Celsius and where the coefficients $b_j$ are empirical parameters. A development at order $M = 4$ is usually sufficient to describe a condensed material over a large temperature range. For water, one has:

\begin{align*}
b_0 &= 1.34359 \\
b_1 &= -1.0514 \times 10^{-4} \\
b_2 &= -1.5692 \times 10^{-6} \\
b_3 &= 5.7538 \times 10^{-9} \\
b_4 &= -1.2873 \times 10^{-11}
\end{align*}

In order to simplify the notations in the following, let us recast Eq. (1) into:

$$\delta n = \sum_{j=0}^{M} \beta_j \delta T^j$$

where

$$\delta n = n(T) - n(T_\infty),$$

$$\delta T = T - T_\infty$$

and where $T_\infty$ is the ambient temperature. A simple calculation yields:

$$\beta_1 = b_1 + 2 T_\infty b_2 + 3 T_\infty^2 b_3 + 4 T_\infty^3 b_4,$$

$$\beta_2 = b_2 + 3 T_\infty b_3 + 6 T_\infty^2 b_4,$$

$$\beta_3 = b_3 + 4 T_\infty b_4,$$

$$\beta_4 = b_4.$$

In this section, we first consider the case of sufficiently small temperature variations such that a linear dependency of the optical index as function of the temperature is a good approximation. In water, this assumption is justified as long as the temperature increase does not exceed $\sim 20$ K above the ambient temperature $T_\infty$. In this case,

$$\delta n = \beta_4 \delta T.$$  (5)

As explained in the article, this thermal-induced variation of the optical index is responsible of the optical path difference (OPD) measured by the wave-front analyzer (WFA). Using the Green’s formalism, the measured OPD distribution $\delta \ell_0(x, y)$ can be linked to the distribution of the heat power density (HPD) $\tilde{p}(x, y)$:

$$\delta \ell_0(x, y) = [\tilde{p} \otimes G_T](x, y).$$  (6)

In order to retrieve the HPD $\tilde{p}(x, y)$, a deconvolution of Eq. (6) is required. This can be simply calculated in the Fourier space where a convolution becomes a simple multiplication:

$$\tilde{p}(k) = \delta \ell_0(k)/\tilde{G}_T(k)$$  (7)

where $\tilde{f}$ is the Fourier transform of $f$. However such a direct deconvolution is likely to cause some divergence issues when the OPD image is noisy. This mathematical problem is well-known and different solutions have been proposed. Among them, one of the simplest solution is the Tikhonov regularization technique. It consists in replacing Eq. (7) by:

$$\tilde{p}(k) = \delta \ell_0(k) \frac{\tilde{G}_T^*(k)}{|\tilde{G}_T(k)|^2 + \alpha}$$

where $z^*$ means the complex conjugate of $z$. The choice of the parameter $\alpha \neq 0$ is empirical. Figure 1 shows its effect and how it can be chosen. If $\alpha$ is too small, the processed OPD image is likely to be very noisy. If $\alpha$ is too large, it tends to damp the processed image, leading to a under-estimation of the HPD. Between these two extreme cases, a proper value of $\alpha$ can be chosen that tends to reduce the noise on the processed HPD without damping it. Experimentally, we found that $\alpha$ should not exceed $10^{-3}$ to ensure a proper estimation of the HPD, as seen in Fig. 1.

Once the HPD has been calculated using the Tikhonov method, we simply work out the temperature distribution, still in the Fourier domain, using the Green’s function $G_T$:

$$\delta \tilde{T}(k) = \tilde{p}(k) \tilde{G}_T(k)$$

followed by an inverse Fourier transform of $\delta \tilde{T}(k)$. Figure 2 summarizes the overall procedure using a block diagram.

B. Non-linear algorithm

In the most general case, i.e. when the temperature increase exceeds $\sim 20$ K (in water), the non-linear (NL)
δℓ(x, y) → Tikhonov deconvolution

\[ \hat{p}(x, y) \] → \[ δT(x, y) \]

FIG. 2. Block diagram describing the numerical procedure used to work out the HPD \( p(x, y) \) and temperature \( T(x, y) \) distributions from the measured OPD distribution \( δℓ(x, y) \).

dependence of the optical index as function of the temperature cannot be discarded. This would rapidly yield an over-estimation of the temperature increase. The Taylor development at least at the fourth order becomes a requirement:

\[ \delta n = \beta_1 \delta T + \beta_2 \delta T^2 + \beta_3 \delta T^3 + \beta_4 \delta T^4 \quad (10) \]

To illustrate the influence of the NL terms, \( \delta n \) has been plotted as a function of \( T \) for Taylor developments at the 1st, 2nd, 3rd and 4th orders in Fig. 3.

Let us first consider the case of a point-like source of heat \( \bar{p}(x, y) = P_0 \delta(x) \delta(y) \). Eq. 9 of the article derived when \( n(T) \) was linear is no longer valid in the present case. Using Eq. (10) and \( \delta T(p, z) = \frac{P_0}{\sqrt{p^2 + z^2}} \), the calculation of the integral \( \delta T(p) = \int_0^h \delta n(p, z) dz \) yields an analytical solution:

\[
\begin{align*}
\delta T(p) &= P_0 \beta_1 \ln(u + \bar{u}) \\
&+ P_0^2 \beta_2 \frac{1}{\rho} \tan^{-1}(u) \\
&+ P_0^3 \beta_3 \frac{1}{\rho^2} \frac{u}{\bar{u}} \\
&+ P_0^4 \beta_4 \frac{1}{\rho^3} \frac{1}{2} \left[ \frac{u}{\bar{u}^2} + \tan^{-1}(u) \right]
\end{align*}
\quad (11)
\]

FIG. 3. Variation of the optical index of water \( \delta n = n(T) - n(25^\circ) \) as function of the temperature. Taylor developments at the 1st, 2nd, 3rd and 4th orders are plotted according to Eq. (10).

where

\[ u = h/\rho \]
\[ \bar{u} = \sqrt{1 + u^2} \]

Eq. (11) can be used to accurately retrieve the heat power \( P_0 \) from the temperature profile generated by the localized source of heat, independently on the temperature range.

FIG. 1. Influence of the choice of the Tikhonov parameter \( \alpha \) illustrated on a gold microstructure featuring 3 rectangle areas composed of arrays of quasi-hexagonally organized gold nanodots. The structure was illuminated at \( \lambda = 532 \text{ nm} \) and \( I = 216 \mu W/\mu m^2 \). a) Thermo-induced optical path difference on a microstructure composed of 3 parallel rectangle areas of gold nanoparticles. b-f) Heat power density distributions computed using different Tikhonov parameters \( \alpha \). g) Optical image of the structures. h-l) Associated HSD profiles average over 16 horizontal lines.
Interestingly,
\[
\ln(u + \bar{u}) \sim -\ln(\rho) \quad \text{as } h \to \infty
\]
\[
\tan^{-1}(u) \to \pi/2 \quad \text{as } h \to \infty
\]
\[
u/\bar{u} \to 1 \quad \text{as } h \to \infty
\]
\[
u/\bar{u}^2 + \tan^{-1}(u) \to \pi/4.
\]

Consequently, when the surrounding liquid layer can be considered as infinitely thick (compared to the extension of the temperature distribution), one ends up with a simpler formula:
\[
\delta\ell (p) = -\bar{P} / \beta_1 \ln(\rho) + \pi \frac{P_0^2 \beta_2}{2 \rho^2} + \frac{P_0^3 \beta_3}{\rho^3} + \pi \frac{P_0^4 \beta_4}{4 \rho^4}
\]

Let us consider now a non-localized source of heat \( \bar{p}(x, y) \). Unfortunately, the simple procedure introduced in the previous section – based on the Green’s formalism and the Tikhonov deconvolution algorithm – is no longer valid if non-linearities occur. To overcome this problem, several deconvolution techniques can be used. We proposed here to use a procedure that relies on an iterative algorithm where the HPD \( \bar{p}(x, y) \) is refined at each loop \( i \). The procedure is schematized in Fig. 4.

In the first loop \( (i = 1) \), a HPD \( \bar{p}_1(x, y) \) is computed from the measured OPD using the linear Tikhonov algorithm. This first estimation of the HPD is then convoluted in 3 dimensions (3D) by the Green’s function \( G_T(r) \) over the whole liquid layer, which leads to a first estimate of the temperature distribution in 3D \( T_1(x, y, z) \) within the whole liquid layer. Then, a first estimate of the 3D profile of the optical index \( \delta n_1(x, y, z) \) is calculated from the temperature profile using its NL expression (Eq. (10)). Finally, by integrating the OPD over the \( z \) direction, one ends up with an OPD map \( \delta\ell_1(x, y) \). In the linear case, one should get \( \delta\ell_1(x, y) \approx \delta\ell_0(x, y) \) (if the damping factor \( g = 1 \)) but the non-linearities generate a discrepancy. For this reason, at each loop, the difference \( \delta\ell_0 - \delta\ell_{i-1} \) is re-injected into the numerical procedure to compute a HPD correction \( \delta p_i \). Then, the corrected HPD \( p_{i-1} + \delta p_i \) is used to compute a new OPD map \( \delta\ell_i \). This procedure can be iterated as many times as desired to get a distribution \( \bar{p}(x, y) = g \sum_{i=1}^{\infty} \delta p_i(x, y) \) that generates a OPD distribution \( \delta\ell_N \) as close as possible to the measured OPD \( \delta\ell_0 \). Note that a damping factor \( g (0 < g \leq 1) \) can be introduced in the procedure. It makes the evolution toward the proper OPD distribution \( \delta\ell_0 \) slower but more accurate and it ensures convergence. \( g \approx 0.3 \) is usually a good compromise.

![FIG. 4. Block diagram describing the numerical procedure used to work out the HPD \( \bar{p}(x, y) \) and temperature \( T(x, y) \) distributions from the measured OPD distribution \( \delta\ell(x, y) \).](image)

In order to illustrate the effect of this NL approach compared to the linear approximation, Fig. 5 plots the evolution of the temperature measured on a given structure as function of the laser power using the linear and NL algorithms. One can see that above a temperature increase of 20 K, it becomes necessary to use the non-linear algorithm. Note that when using the NL algorithm, one does observe a linear variation of the temperature increase as function of the laser power, which is the expected behavior.

![FIG. 5. Average temperature increase of a plus-shape structure as function of the laser irradiance. Blue crosses (x): Artefactual curve obtained when not using the NL algorithm, leading to an overestimation of the temperature. Red circles (○): Proper and expected linear dependence of the temperature increase as function of the laser irradiance obtained when using the NL algorithm, supporting his validity.](image)

Using this iterative algorithm is the choice we have done for the sake of simplicity. However, it could be interesting in the future to investigate other possible optimization techniques based on the minimization of an approximation criterion such that \( \int \left[ \delta\ell(x, y) - \int_0^h \delta n dx dy \right]^2 \).

C. Principle of wavefront analysis using quadri-wave lateral shearing interferometry

The thermal measurements presented in the article are based on quadriwave lateral shearing interferometry (QWLSI) measurements. In this section, we wish to give a brief description of the principle of this technique. For more details, we invite the reader to refer to Refs. [3],[4]
One-dimensional lateral shearing interferometry is a technique aiming at measuring the phase gradient of a given light wavefront in one direction from the interference between two identical but tilted copies of the wavefront. Such an approach does not require the use of any reference light beam, which is a crucial advantage. The replication of the incident wavefront is achieved using a sinusoidal phase grating (Fig. 6). After a few millimeters of propagation, the mutual interference pattern is recorded with a CCD camera. In the simplest case of a flat incident wavefront, the interference pattern consists of regular fringes featuring a period $\Lambda$ (Fig. 6a), leading to two well-defined spots in the Fourier space at the spatial frequencies $\pm 1/\Lambda$ (Fig. 6b). In the case of an arbitrarily distorted wavefront, a local phase gradient leads to a local variation of the fringe frequency. In practice, phase gradients are recovered in the Fourier space by means of a deconvolution around the nominal interferogram fringe frequency $1/\Lambda$ (Fig. 6d).

In order to obtain two-dimensional (2D) images, a 2D phase grating has to be used (Fig. 7). Such an approach, called quadrilinear lateral shearing interferometry (QWLSI), leads to four replicas of the incident wavefront that interfere all together. The interferogram is recorded by a CCD. The two processed gradient profiles along two crossed directions are then integrated to determine both the intensity and the phase of the incident field by deconvolution around the frequency $1/\Lambda$. As for one-dimensional lateral shearing interferometry, one could use a sinusoidal two-dimensional phase grating to generate the 4 replicated wavefronts required by QWLSI (Fig. 7a). In practice, the so-called Modified Hartmann Grating (MHG) is preferably used because of its much simpler fabrication process: MHG, as shown in Fig. 7b, is an only 3-level amplitude component that has been optimized by properly choosing the ratio between the size of the square holes and the pitch of the grating to diffract more than 90% of the light energy into the 4 first orders required in QWLSI.

The intensity distribution on the CCD sensor can be calculated from the coherent addition of the four displaced replicas. Assuming a paraxial propagation and neglecting free space diffraction, the intensity distribution reads:

\[
I(x, z) = I_0 \left[ 1 + \cos \left( \frac{2\pi}{\Lambda} x + \frac{2\pi}{\Lambda} \frac{\partial \delta \ell}{\partial x} \right) + \cos \left( \frac{2\pi}{\Lambda} y + \frac{2\pi}{\Lambda} \frac{\partial \delta \ell}{\partial y} \right) 
+ \frac{1}{2} \left( \cos \left( \frac{2\pi}{\Lambda} (x + y) + \frac{2\pi}{\Lambda} z \frac{\partial \delta \ell}{\partial (x + y)} \right) + \cos \left( \frac{2\pi}{\Lambda} (x - y) + \frac{2\pi}{\Lambda} z \frac{\partial \delta \ell}{\partial (x - y)} \right) \right) \right]
\]

where $z$ is the propagation length along the optical axis, and $I_0$ is the maximum intensity of the incident light at
the entrance of the MHG \((z = 0)\). This equation shows that both the interferogram period and the modulation due to the OPD are wavelength-independent. The use of a temporally incoherent broad-band source (as a halogen lamp) is thus possible.\(^5,6\) As described in the paper, we directly benefited from such an achromatic property by simply using the native broadband halogen source of the system to measure thermo-induced refractive index changes in the sample.

Because phase information is coded in the interferogram by a spatial frequency modulation, phase and intensity of the incident light beam are independently determined. The price for this is that the resulting phase image is 4 times smaller in lateral size than the original interferogram image recorder onto the CCD. Indeed, 4×4 pixels of the CCD are used to process one pixel of the phase image. However, the advantage is that this CCD-based approach provides single-shot quantitative phase imaging and thus high speed and high resolution measurements.

\(^{2}\) A. N. Tikhonov, Solution of Ill-posed problems (Vh Winston, 1977).